Design and Validation of a Couette Flow Viscometer

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Franklin W. Olin College of Engineering: Introduction to Thermal-Fluid Systems

BACKGROUND

Viscosity is a critical fluid property for industrial applications from oil pipelines to pharmaceutical delivery systems. Traditional viscometers use rotational shear, gravitational flow, or falling ball methods to measure viscosity. We designed a sticky piston viscometer based on Couette flow, which is shear between two surfaces moving relative to each other. This simple apparatus directly measures viscous forces using an Instron tensile tester.

SYSTEM DESCRIPTION

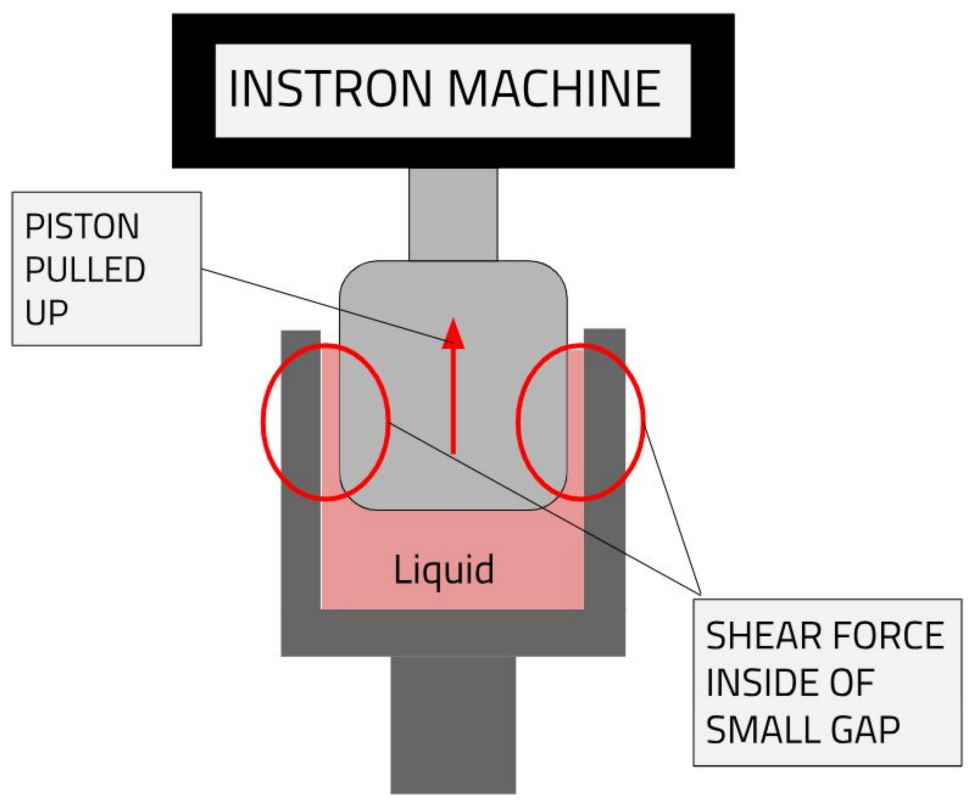

Figure 1: System diagram featuring Instron, translating piston, bottom container, and shear force interaction

System: A delrin piston is pulled upward from an acrylic container filled with viscous liquid. The narrow 0.25 mm gap between the piston and container creates shear flow as the piston moves.

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Interactions with surroundings:

-The piston surface applies shear stress to the fluid through viscous friction

-Air enters through central drilled hole in the piston to equalize pressure

Measurement interaction: An Instron tensile testing machine pulls the piston upward at constant velocity (v = 0.0085 m/s) while measuring the resistive force.

MODEL & PREDICTION

Governing equation: The shear stress on the piston is modeled using Newton's law of viscosity for Couette flow in a thin annular gap:

- τ = Shear Stress [Pa]
- μ = Dynamic Viscosity [Pa·s]
- v = piston velocity [m/s]
- δ = gap width [m]
- R = Piston Radius [m]
- H = Immersed Height [m]
- F = Force on Piston [N]

$$F = \tau \times A = \frac{2\pi R\mu v}{\delta} \times h$$

Material properties used:

- Karo Light Corn Syrup Dynamic Viscosity: μ = 2 Pa·s @ 20°C
- Glycerin Dynamic Viscosity: μ = 1.41 Pa·s @ 20°C
- Gap width: δ = .25mm

Initial condition: h(t = 0) = 80mm (piston fully immersed at start of data collection

Boundary conditions: Piston Moves at constant velocity (8.5mm/s upwards

Solution method: The force is calculated analytically using the governing equation. As the piston rises, the immerse height decreases linearly with time according to $h(t) = h_0 - vt$. The predicted force decreases linearly with immersed height, creating a linear relationship: F(h) = (2πRμv/δ) · h. This allows for direct calculation of viscosity simply from the measured slope of force versus height.

Key assumptions:

1. **Uniform Gap Width (δ = 0.25 mm):** Piston OD and Container ID are perfectly .25mm away from each other.
2. **No Suction Effects:** Central hole through the center of piston allows for unrestricted air entry, ensuring all recorded force is shear force.
3. **Negligible Drainage due to Gravity:** Glycerin & Corn Syrup are too viscous to drain downwards and reduce contact surface area during pulling of piston
4. **No-slip Boundary Condition:** Fluid velocity at container wall & piston surface = 0 m/s everywhere.
5. **Negligible Kinetic Energy Affects + Energy Dissipation:** Pulling Velocity is slow enough that inertial/acceleration effects do not affect the steady state of the flow.
6. **Rigid Body Motion:** The system is made of various 3D printed, plastic, and metal components with 0 deformation as a result of the pulling of the Instron.
7. **Laminar Flow is maintained:** Reynolds number is < 1, so flow remains orderly, not turbulent. P = density of glycerin = 1.26g/cm^3 → Re = 1.9 · 10^3

$$Re = \frac{\rho v \delta}{\mu}$$

REFLECTION

What Surprised Us:

We were surprised that even a ¼" diameter hole was insufficient for air entry and required 2 extra ⅓" holes to ensure air could move without restriction. Another surprising thing is that the plateau, although disappointing, was a clue and helped us find why it was behaving that way.

If we were to do this project over again, we would...

Test the venting design with water/cheap liquid before using comparatively costly glycerine. We would also start with larger numerous air entry holes.

EXPERIMENTAL RESULTS

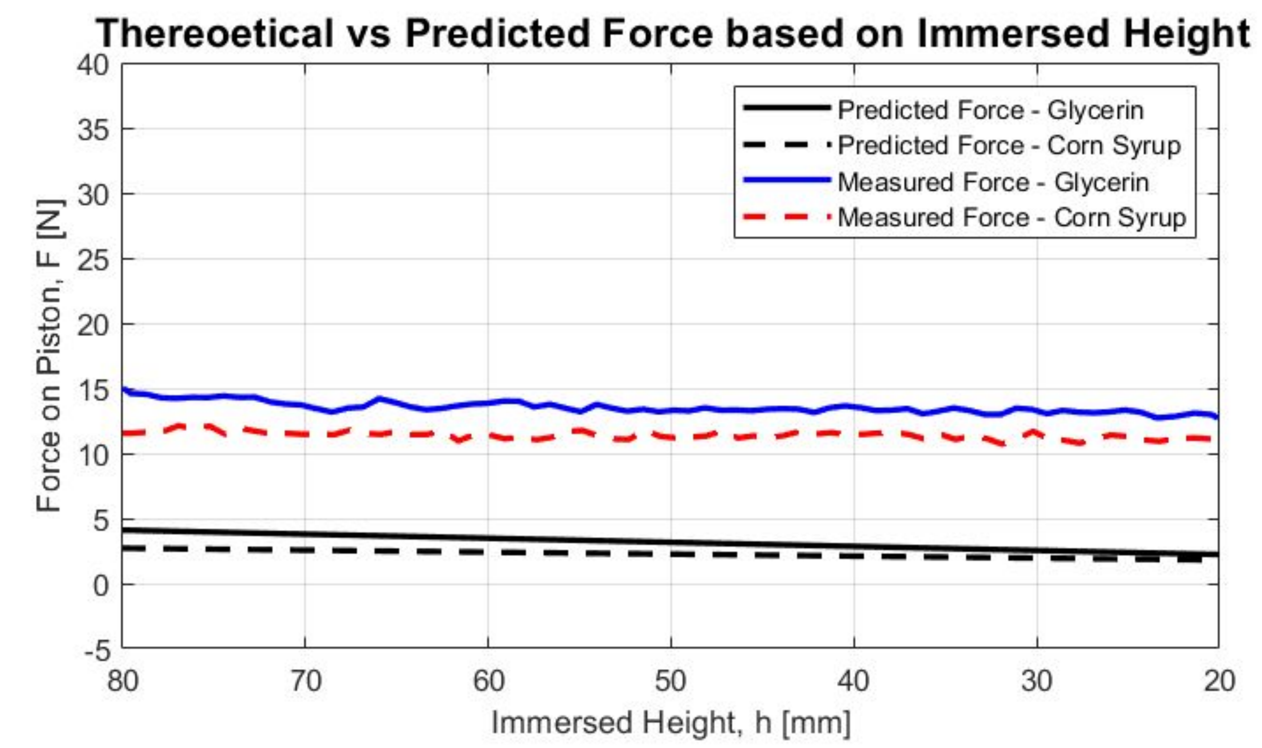

Figure 2: Comparison of predicted (black) and measured (blue/red) Forces during 60mm travel. The 'Measured Force's are an average of 4 separate data collections each. R^2 value for Glycerin is .08. R^2 value for Corn Syrup is .09.

CONCLUSION

Our model and apparatus proved successful after modifications, demonstrating the predicted linear decline in measured force as immersed height decreased. It is possible to create a viscometer using various pieces of stock.

Assumptions that held valid

Laminar Flow: Confirmed by the linear height→ force relationship

Uniform Gap Width: Supported by consistent results across numerous trials.

Rigid Body Motion: Assembly held steady for all tests and is ready for more.

Negligible Kinetic Energy Effects + Energy Dissipation: slow pull speed of 8mm/s ensured liquid was always in steady state.

Negligible Drainage: Even after almost 2 dozen tests, no visual signs of liquids draining too quickly.

Assumption that did not hold valid

No Suction Effects: There were in fact large suction effects found in our set up. Upon diagnosis, extra holes were drilled to ensure air could escape without obstruction during tests and suction had no chance to act.

ANALYSIS

Initial Attempt: The model predicted force should decrease linearly as immersed height decreased from 80mm to 20mm, yet experimental data showed force remained near constant. As surface area decreased, shear force should have decreased with height, but it did not and hovered around 12-14N.

Investigation Process: To determine whether the discrepancy was related to a fault in our model, our physical system, or assumptions, we explored various avenues. First, we tested Corn Syrup and observed identical plateauing behavior, confirming that it was not as simple as glycerin being not viscous enough. We systematically discussed and ruled out hypotheses for the disconnect between our model and measured data. The Couette flow model is well researched and established, so we knew our model was sound. The weight contribution of the piston was confirmed negligible when tested without any fluid. We tested with diluted glycerin solutions but not one trial showed any decline in measured force. We confirmed that the gap width, δ, was consistently .25mm.

A turning point came when we held our apparatus upside down. If the central hole truly allowed full air entry and escape, then the piston should fall out under its own weight, but it took .75 seconds to drop out. Completing some back-of-the-napkin calculations using the mass of the piston revealed that the force of suction contributed to at least 2.5N of force, resulting in a .75 second drop time.

$$\frac{2d}{t^2} = a \qquad a = \frac{2(.09)}{.75^2} = .32 \qquad \sum F = ma \qquad mg - F_{suction} = ma$$

$$F_{suction} = 2.55N$$

This demonstrated significant air resistance (2.55N), proving our assumption of proper air flow through the central hole incorrect.

So, when the Instron was measuring the force, it was actually measuring Total = $F_{Shear}(h) + F_{Suction}$. Since $F_{Suction}$ is dependent on velocity but not height, it masks the expected linear decrease in F_{Shear}.

To test this hypothesis, we drilled two additional wide holes through the piston to improve air entry and escape. Although we had no remaining glycerin, we tested with corn syrup and found the measured force now decreased linearly with height, matching the model prediction. This confirmed inadequate venting as the root cause.

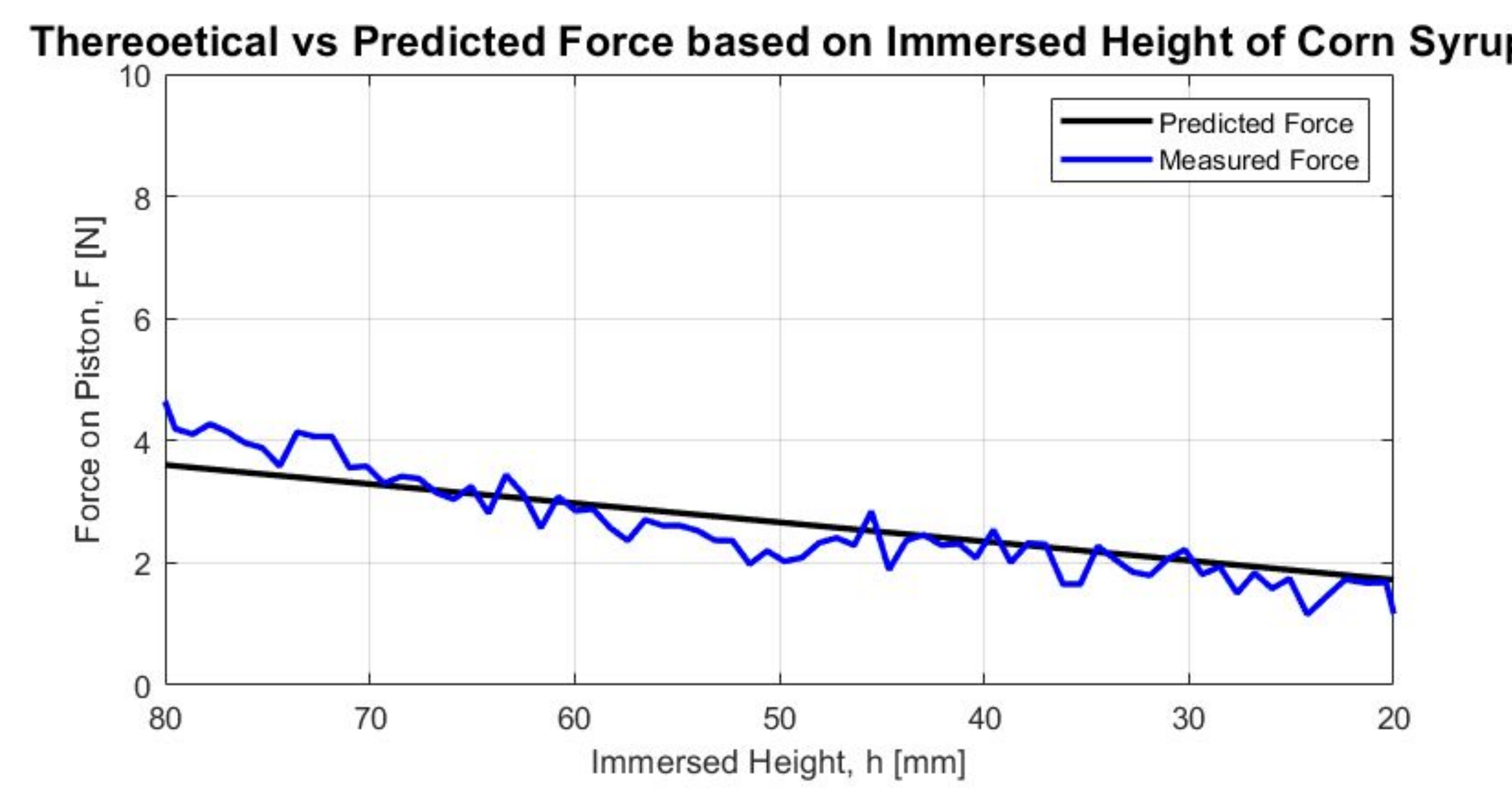

Figure 3: Comparison of predicted (black) and measured (blue) Forces during 60mm travel. The 'Measured Force's are an average of 4 separate data collections each. This series of tests were conducted after drilling large holes through the piston, ensuring easy air entry and leaving. This series of tests resulted in a near perfect match (R^2 = .88). The high value shows that the predicted line closely matches the behavior of the measured force. This is a great improvement from the previous value of .09.